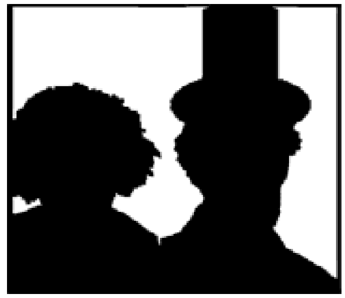
9th Midwest Relativity Meeting at the University of Illinois at Urbana-Champaign - November 12 & 13, 1999



Traversable wormholes from massless conformally couple scalar fields

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ABSTRACT

The massless conformally coupled scalar field is characterized by the so-called "new improved stress-energy tensor", which is capable of classically violating the null energy condition. When coupled to Einstein gravity we find a three-parameter class of exact solutions. These exact solutions include the Schwarzschild geometry, assorted naked singularities, and a large class of traversable wormholes.

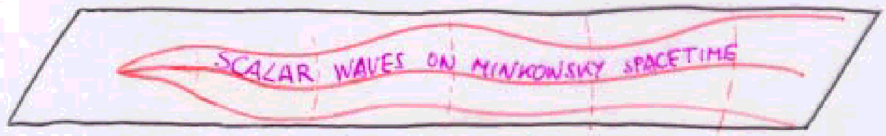
TRAVERSABLE WORMHOLES FROM MASSLESS CONFORMALLY COUPLED SCALAR FIELDS

CARLOS BARCELÓ & MATT VISSER

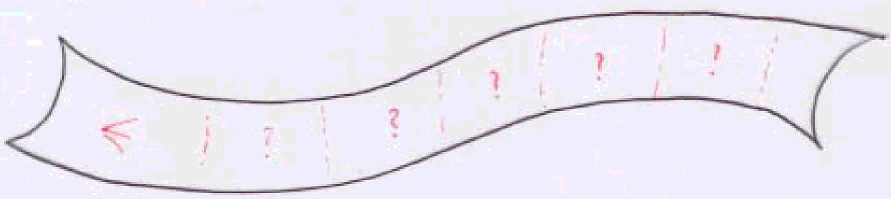
gr-qc/9908029 TO APPEAR IN PLB

SCALAR FIELDS ON CURVED SPACETIMES

SUPPOSE THAT WE KNOW THE BEHAVIOUR OF A SCALAR FIELD ON A MINKOWSKI SPACETIME



WHAT IS ITS BEHAVIOUR WHEN THE SPACETIME IS CURVED?



NO UNIQUE PRESCRIPTION!

- MINIMUM COUPLING
  - MINIMAL SUBSTITUTION $(\partial_\rho \longrightarrow \nabla_\rho)$
- CONFORMAL COUPLING
  - A POTENTIAL TERM ADDED $\left(\mathcal{L}_c(\phi) = \mathcal{L}_M(\phi) - \frac{\sqrt{-g}}{12} R \phi^2\right)$
- OTHERS
  - $\mathcal{L}_\xi(\phi) = \mathcal{L}_M(\phi) - \frac{\sqrt{-g}}{2} \xi R \phi^2 \qquad 0 \neq \xi \neq \frac{1}{6}$
  - ⋮

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* WE WILL RESTRICT FOR SIMPLICITY TO A FREE A MASSLESS SCALAR FIELD

CONFORMAL COUPLING

- KEEPS THE CONFORMAL INVARIANCE
  - MIMICS THE PHOTONS BEHAVIOR
- ITS ENERGY-MOMENTUM TENSOR YIELD THE "NEW IMPROVED ENERGY-MOMENTUM TENSOR" OF PARTICLE PHYSICS WHEN PARTICULARIZED TO FLAT SPACETIME.
  - FINITE MATRIX ELEMENTS IN RENORMALIZED PERTURBATION THEORY.
  - NATURAL BEHAVIOUR AT LOW ENERGIES

SCALAR FIELD AS THE SOURCE OF CURVATURE

MINIMAL COUPLING $\longmapsto$ CONFORMAL COUPLING

CLASSICAL SOLUTIONS GRAVITY + MINIMALLY COUPLED SCALAR $\longmapsto$ CLASSICAL SOLUTIONS GRAVITY + CONFORMALLY COUPLED SCALAR

POINT WISE AND AVERAGED NULL ENERGY CONDITIONS

EINSTEIN EQUATIONS

$$\kappa G_{\mu\nu} = T_{\mu\nu}(\phi, \cancel{g}_{\mu\nu}) \qquad \kappa = \frac{1}{8\pi G}$$

LET $x^\mu(\lambda)$ BE A LIGHT LIKE CURVE WITH TANGENT VECTOR $k^\mu = \frac{dx^\mu}{d\lambda}$

$$k^\mu k^\nu T_{\mu\nu}(\phi) = (1 - \phi^2/6\kappa)^{-1}\left[\frac{2}{3}\left(\frac{d\phi}{d\lambda}\right)^2 - \frac{1}{3}\phi\frac{d^2\phi}{d\lambda^2}\right]$$

CAN BE NEGATIVE!

ALSO $\int d\lambda\, k^\mu k^\nu T_{\mu\nu}(\phi)$ CAN BE NEGATIVE!

IT CAN BE SITUATIONS IN WHICH EVEN A NULL GEODESIC OBSERVER WILL SEE NEGATIVE ENERGY DENSITIES

SO...

CAN A CONFORMAL SCALAR FIELD SUPPORT TRAVERSABLE WORMHOLE GEOMETRIES?

STATIC AND SPHERICALLY SYMMETRIC SOLUTIONS

EQUATIONS:

1) $R=0$

2) $\kappa G_{00} = T_{00}(\phi)$

3) $\Box\phi = 0$

MINIMALLY COUPLED SCALAR FIELD SOLUTIONS (JNWW)

$$ds_M^2 = -\left(1-\frac{2\eta}{r}\right)^{\cos\chi} dt^2 + \left(1-\frac{2\eta}{r}\right)^{-\cos\chi} dr^2 + \left(1-\frac{2\eta}{r}\right)^{1-\cos\chi} r^2 d\Omega_2^2$$

$$\phi_M = \sqrt{\frac{\kappa}{2}} \sin\chi \ln\left(1-\frac{2\eta}{r}\right)$$

LET NOW $ds_C^2 = \Omega^2(r)\, ds_M^2$

$$\underbrace{\text{EQUATIONS } 1,2,3}$$

$$\Omega = \alpha_+ \exp(+\phi_M/\sqrt{6\kappa}) + \alpha_- \exp(-\phi_M/\sqrt{6\kappa})$$

$$\phi_c = \pm\sqrt{6\kappa}\left[\frac{\alpha_+ \exp(+\phi_M/\sqrt{6\kappa}) - \alpha_- \exp(-\phi_M/\sqrt{6\kappa})}{\alpha_+ \exp(+\phi_M/\sqrt{6\kappa}) + \alpha_+ \exp(-\phi_M/\sqrt{6\kappa})}\right]$$

GENERALIZATION OF THE SOLUTIONS $\begin{cases} \text{FROYLAND} \\ \text{AGNESE \& LA CAMERA} \end{cases}$

PARAMETER SPACE

DEFINITION: $\tan(\Delta/2) = \dfrac{\alpha_+ - \alpha_-}{\alpha_+ + \alpha_-}$

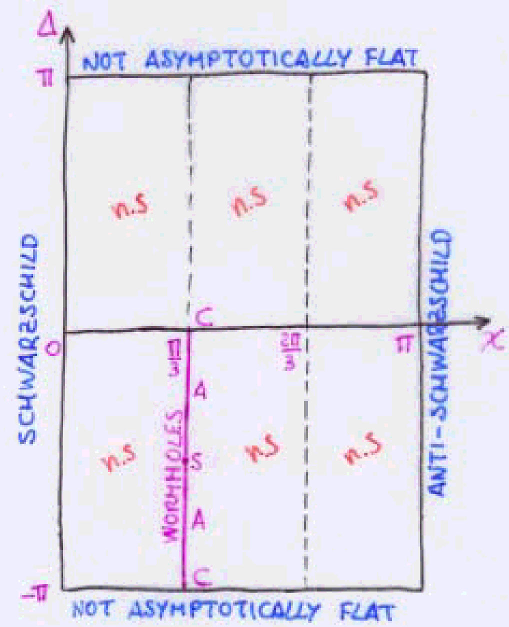
- THREE PARAMETER FAMILY OF SOLUTIONS

  $\eta \; ; \; \chi \in (-\pi, \pi] \; ; \; \Delta \in (-\pi, \pi)$

  RELATED WITH: M, Q, ϕ_∞

- AFTER SOME IDENTIFICATIONS $\begin{cases} (\eta, \chi, \Delta) \cong (\eta, -\chi, -\Delta) \\ (\eta, \chi, \Delta) \cong (-\eta, \chi+\pi, \Delta) \end{cases}$

  WE ONLY HAVE TO DEAL WITH $\eta \geq 0$, $\chi \in [0, \pi]$, $\Delta \in (-\pi, \pi]$



TRAVERSABLE WORMHOLES GEOMETRIES

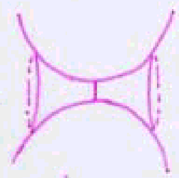
BY USING ISOTROPIC COORDINATES $r = \bar{r}\left(1 + \frac{\gamma}{2\bar{r}}\right)^2$

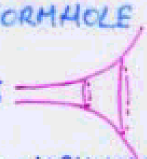
WE CAN WRITE:

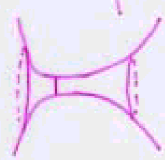
$$ds_c^2 = \Big[\underbrace{\alpha_+ \frac{\left(1 - \frac{\gamma}{2\bar{r}}\right)}{\left(1 + \frac{\gamma}{2\bar{r}}\right)} + \alpha_-}_{\equiv\ \Omega'(\bar{r})}\Big]^2 \left[-dt^2 + \left(1 + \frac{\gamma}{2\bar{r}}\right)^4 \left(d\bar{r}^2 + \bar{r}^2 d\Omega_2^2\right)\right]$$

FOR $\frac{\alpha_+ - \alpha_-}{\alpha_+ + \alpha_-} < 0 \Rightarrow$ WORMHOLE GEOMETRIES : $\Omega'(\bar{r}) \neq 0 \ \forall \bar{r} \in [0, +\infty)$

$$\phi = \pm\sqrt{6\kappa}\ \frac{\alpha_+\left(1 - \frac{\gamma}{2\bar{r}}\right) - \alpha_-\left(1 + \frac{\gamma}{2\bar{r}}\right)}{\alpha_+\left(1 - \frac{\gamma}{2\bar{r}}\right) + \alpha_-\left(1 + \frac{\gamma}{2\bar{r}}\right)}$$

$\alpha_+ = 0 \Rightarrow$ SYMMETRIC WORMHOLE  $\phi = \pm\sqrt{6\kappa}$

$\alpha_+ = \pm\alpha_- \Rightarrow$ CORNUCOPIAS 

THE REST $\Rightarrow$ ASYMMETRIC WORMHOLES 

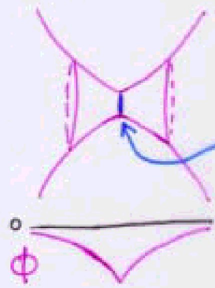
IN ALL THESE WORMHOLES $G_{eff} = G\left(1 - \phi^2/6\kappa\right)^{-1}$

HAS A DIFFERENT SIGN ON EACH

ASYMPTOTIC REGION

WORK IN PROGRESS

- BUILDING "SYMMETRIC" WORMHOLES



THIN SHELL OF NORMAL MATTER

$$T_{\mu\nu} = \delta(\bar{r}-\bar{r}_0)\, S_{\mu\nu}$$

- GENERALIZATION TO ARBITRARY ξ COUPLINGS
    - HOW THE VALUE $\xi = \frac{1}{4}$, THRESHOLD FOR ANEC VIOLATIONS, SHOWS UP ON THESE CONSTRACTIONS?